Paper Reference(s)

9801/01

Edexcel

Mathematics

Advanced Extension Award

Monday 27 June 2011 – Afternoon

Time: 3 hours

Materials required for examination	**Items included with question papers**
Answer book (AB16)	Nil
Graph paper (ASG2)	
Mathematical Formulae (Pink)	

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, initials and signature.
Check that you have the correct question paper.
Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 100, of which 7 marks are for style, clarity and presentation.
There are 8 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Solve for $0 \leqslant \theta \leqslant 180°$

$$\tan\left(\theta + 35°\right) = \cot\left(\theta - 53°\right)$$

(Total 4 marks)

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2. Given that

$$\int_0^{\frac{\pi}{2}} \left(1 + \tan\left[\tfrac{1}{2}x\right]\right)^2 \, dx = a + \ln b$$

find the value of a and the value of b.

(Total 7 marks)

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3. A sequence $\{u_n\}$ is given by

$$\begin{aligned} u_1 &= k & & \\ u_{2n} &= u_{2n-1} \times p & & n \geqslant 1 \\ u_{2n+1} &= u_{2n} \times q & & n \geqslant 1 \end{aligned}$$

where k, p and q are positive constants with $pq \neq 1$

(a) Write down the first 6 terms of this sequence.

(3)

(b) Show that $\displaystyle\sum_{r=1}^{2n} u_r = \frac{k\left(1+p\right)\left(1-(pq)^n\right)}{1-pq}$

(6)

In part (c) $[x]$ means the integer part of x, so for example $[2.73] = 2$, $[4] = 4$ and $[0] = 0$

(c) Find $\displaystyle\sum_{r=1}^{\infty} 6 \times \left(\frac{4}{3}\right)^{\left[\frac{r}{2}\right]} \times \left(\frac{3}{5}\right)^{\left[\frac{r-1}{2}\right]}$

(4)

(Total 13 marks)

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4. The curve C has parametric equations

$$x = \cos^2 t$$

$$y = \cos t \sin t$$

where $0 \leqslant t < \pi$

(a) Show that C is a circle and find its centre and its radius.

(5)

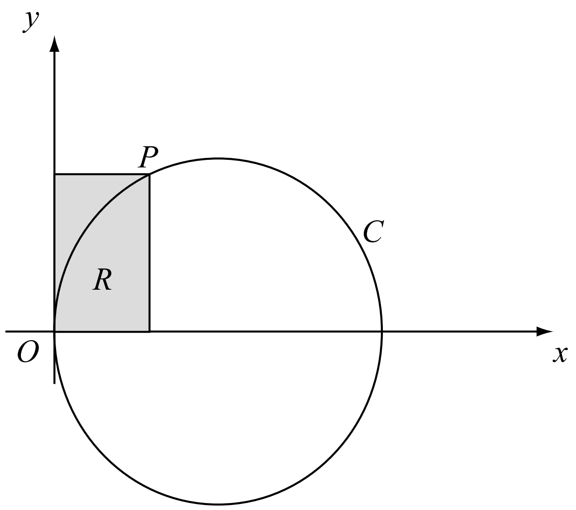

Figure 1

Figure 1 shows a sketch of C. The point P, with coordinates $\left(\cos^2\alpha\,,\ \cos\alpha\sin\alpha\right)$, $0 < \alpha < \frac{\pi}{2}$, lies on C. The rectangle R has one side on the x-axis, one side on the y-axis and OP as a diagonal, where O is the origin.

(b) Show that the area of R is $\sin\alpha\cos^3\alpha$

(1)

(c) Find the maximum area of R, as α varies.

(7)

(Total 13 marks)

5.

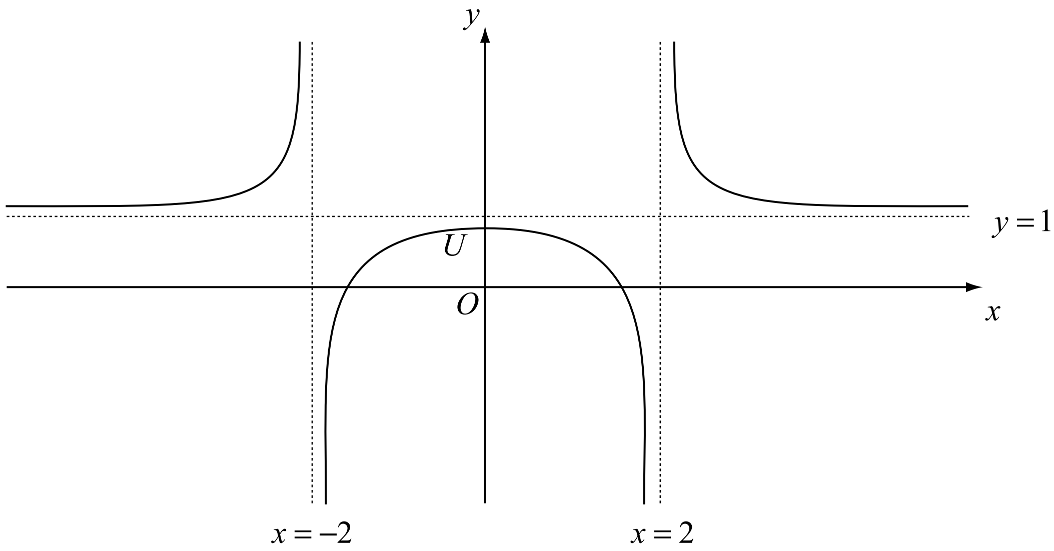

Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \dfrac{x^2 - 2}{x^2 - 4}$ and $x \neq \pm 2$.

The curve cuts the y-axis at U.

(a) Write down the coordinates of the point U.

(1)

The point P with x-coordinate a $(a \neq 0)$ lies on C.

(b) Show that the normal to C at P cuts the y-axis at the point

$$\left(0\ ,\ \left[\frac{a^2 - 2}{a^2 - 4} - \frac{\left(a^2 - 4\right)^2}{4}\right]\right)$$

(6)

The circle E, with centre on the y-axis, touches all three branches of C.

(c) (i) Show that

$$\left[\frac{a^2}{2(a^2 - 4)} - \frac{(a^2 - 4)^2}{4}\right]^2 = a^2 + \frac{(a^2 - 4)^4}{16}$$

(ii) Hence, show that

$$\left(a^2 - 4\right)^2 = 1$$

(iii) Find the centre and radius of E.

(10)

(Total 17 marks)

6. The line L has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ -3 \\ -8 \end{pmatrix} + t\begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix}$$

The point P has position vector $\begin{pmatrix} -7 \\ 2 \\ 7 \end{pmatrix}$.

The point P' is the reflection of P in L.

(a) Find the position vector of P'.

(6)

(b) Show that the point A with position vector $\begin{pmatrix} -7 \\ 9 \\ 8 \end{pmatrix}$ lies on L.

(1)

(c) Show that angle $PAP' = 120°$.

(3)

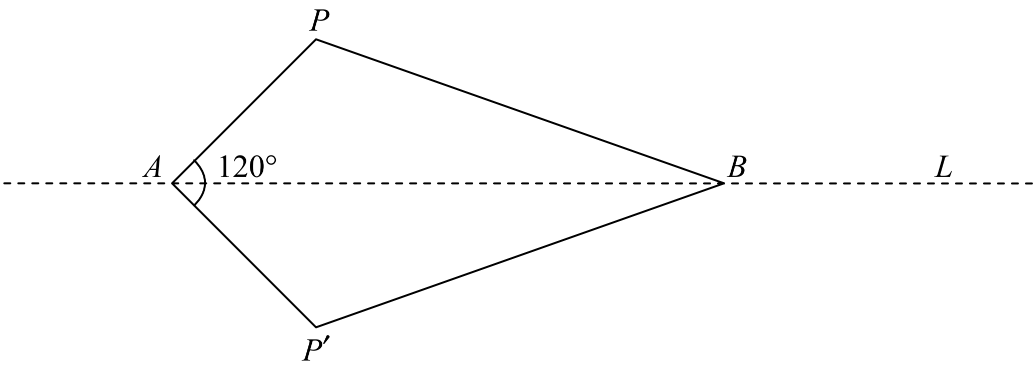

Figure 3

The point B lies on L and $APBP'$ forms a kite as shown in Figure 3.

The area of the kite is $50\sqrt{3}$

(d) Find the position vector of the point B.

(5)

(e) Show that angle $BPA = 90°$.

(2)

The circle C passes through the points A, P, P' and B.

(f) Find the position vector of the centre of C.

(2)

(Total 19 marks)

7.

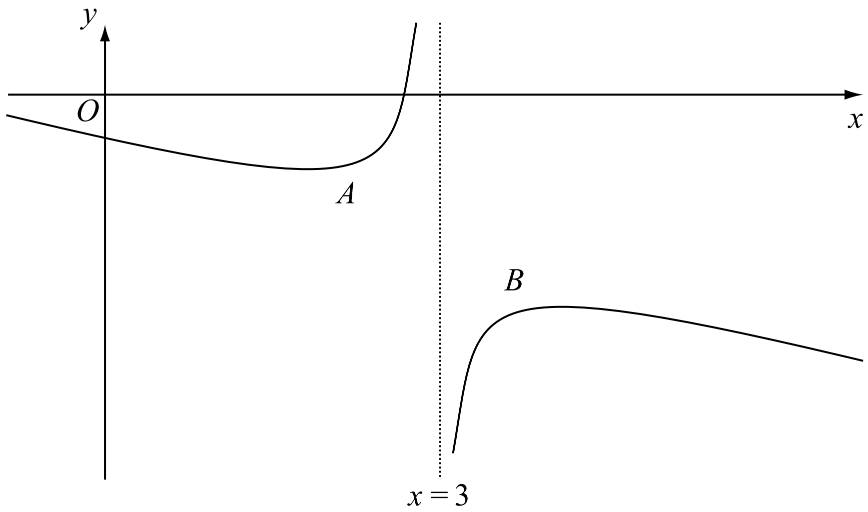

Figure 4

(a) Figure 4 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{x^2 - 5}{3 - x}, \quad x \in \mathbb{R},\ x \neq 3$$

The curve has a minimum at the point A, with x-coordinate α, and a maximum at the point B, with x-coordinate β.

Find the value of α, the value of β and the y-coordinates of the points A and B.

(5)

(b) The functions g and h are defined as follows

$$\mathrm{g}: x \rightarrow x+p \qquad x \in \mathbb{R}$$

$$\mathrm{h}: x \rightarrow |x| \qquad x \in \mathbb{R}$$

where p is a constant.

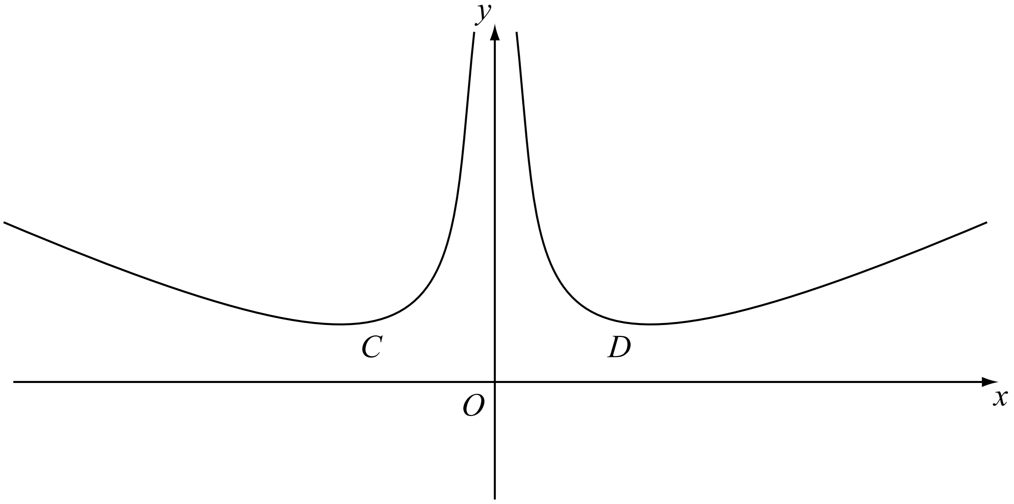

Figure 5

Figure 5 shows a sketch of the curve with equation $y=\mathrm{h}(\mathrm{fg}(x)+q)$, $x \in \mathbb{R}$, $x \neq 0$, where q is a constant. The curve is symmetric about the y-axis and has minimum points at C and D.

(i) Find the value of p and the value of q.

(ii) Write down the coordinates of D.

(5)

(c) The function m is given by

$$\mathrm{m}(x)=\frac{x^2-5}{3-x}, \quad x \in \mathbb{R}, x \leqslant \alpha$$

where α is the x-coordinate of A as found in part (a).

(i) Find m^{-1}

(ii) Write down the domain of m^{-1}

(iii) Find the value of t such that $\mathrm{m}(t)=\mathrm{m}^{-1}(t)$

(10)

(Total 20 marks)

FOR STYLE, CLARITY AND PRESENTATION: 7 MARKS
TOTAL FOR PAPER: 100 MARKS

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